

## Causal queries and causal types.

Examples of models, queries, and the collection of types that satisfy the queries.

Model	Query	Given	Interpretation	Types
$X \rightarrow Y$	$Y[X = 1] > Y[X = 0]$		Probability that X has a positive effect on Y	$X0.Y01, X1.Y01$
$X \rightarrow Y$	$Y[X = 1] < Y[X = 0]$	$X = 1$	Probability that X has a negative effect on Y among those for whom $X=1$	$X1.Y10$
$X \rightarrow Y$	$Y[X = 1] > Y[X = 0]$	$X = 1, Y = 1$	Probability that $Y=1$ is due to $X=1$ (Attribution)	$X1.Y01$
$X \rightarrow Y \leftarrow W$	$Y[X = 1] > Y[X = 0]$	$W = 1$	Probability that X has a positive effect on Y for a case in which $W = 1$ (where W is possibly defined post treatment)	$W1.X0.Y0001, W1.X1.Y0001, W1.X0.Y1001, W1.X1.Y1001, W1.X0.Y0011, W1.X1.Y0011, W1.X0.Y1011, W1.X1.Y1011$
$X \rightarrow Y \leftarrow W$	$Y[X = 1, W = 1] > Y[X = 0, W = 1]$		Probability that X has a positive effect on Y if W were set to 1 for cases for which in fact $W=0$	$W0.X0.Y0001, W0.X1.Y0001, W0.X0.Y1001, W0.X1.Y1001, W0.X0.Y0011, W0.X1.Y0011, W0.X0.Y1011, W0.X1.Y1011$
$X \rightarrow Y \leftarrow W$	$Y[X = 1] > Y[X = 0]$	$Y[W = 1] > Y[W = 0]$	Probability that X has a positive effect on Y for a case in which W has a positive effect on Y	$W0.X0.Y0110, W1.X1.Y0001, W1.X1.Y1001, W0.X0.Y0111$
$X \rightarrow Y \leftarrow W$	$(Y[X = 1, W = 1] > Y[X = 0, W = 1]) > (Y[X = 1, W = 0] > Y[X = 0, W = 0])$	$W = 1, X = 1$	Probability of a positive interaction between W and X for Y; the probability that the effect of X on Y is stronger when W is larger	$W1.X1.Y0001, W1.X1.Y1001, W1.X1.Y1011$
$X \rightarrow M \rightarrow Y \leftarrow X$	$Y[X = 1, M = M[X = 1]] > Y[X = 0, M = M[X = 1]]$	$X = 1, M = 1, Y = 1$	The probability X would have a positive effect on Y if M were controlled to be at the level it would take if X were 1 for units for which in fact $M==1$	$X1.M01.Y0001, X1.M11.Y0001, X1.M01.Y1001, X1.M11.Y1001, X1.M01.Y0101, X1.M11.Y0101, X1.M01.Y1101, X1.M11.Y1101$
$X \rightarrow M \rightarrow Y \leftarrow X$	$(Y[M = 1] > Y[M = 0]), (M[X = 1] > M[X = 0])$	$Y[X = 1] > Y[X = 0], M = 1$	The probability that X causes M and M causes Y among units for which $M = 1$ and X causes Y	$X1.M01.Y0001, X1.M01.Y0011$